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A dynamic game theoretic framework for process plant strategic upgrade and production planning

Philip Tominac and Vladimir Mahalec*

Department of Chemical Engineering, McMaster University, 1280 Main St. W., Hamilton, ON,

L8S 4L8, Canada

*Corresponding author. Tel.: +1 905 525 9140 ext. 26386. E-mail address:

mahalec@mcmaster.ca

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Abstract

A dynamic potential game theoretic production planning framework is presented in which production plants are treated as individual competing entities and competition occurs dynamically over a discrete finite time horizon. A modified Cournot oligopoly with sticky prices provides the basis for dynamic game theoretic competition in a multi-market nonlinear and nonconvex production planning model wherein market price adapts to a value that clears cumulative market supply. The framework is used to investigate a petrochemical refining scenario in which a single inefficient refiner faces elimination by its competitors; we

demonstrate that there exist conditions under which the threatened refiner may upgrade itself in order to become competitive and escape the threat, or alternatively in which the threat of elimination will never be carried out and the refiner is effectively safe in the given market configuration. Globally optimal dynamic Nash equilibrium production trajectories are presented for each case.

Introduction

We develop a dynamic game theoretic framework for strategic production planning and re-examine the static game theoretic competitor elimination problem introduced by Tominac and Mahalec¹. In this problem a plant can be forced to shut down through the actions of its competitors and its market share be claimed by them at the expense of decreased market prices. Extension of optimal game theoretic production planning to the dynamic domain facilitates the analysis of decision-making processes in time-varying competitive markets. In particular, we investigate the elimination problem in a petrochemical refining setting from the perspective of the threatened refiner and the decisions it faces as it attempts to prolong its survival. Game theory provides a means of analyzing the complex interactions between competing and cooperating entities. A number of authors have used game theoretic principals to examine cooperative and competitive interactions in chemical engineering supply chain literature. Gjerdrum, Shah, and Papageorgiou examine fair profit allocation among supply chain participants using transfer prices and Nash bargaining equilibria^{2,3}. Bai, Ouyang, and Pang incorporate agricultural decisions into the biorefinery supply chain using a Stackelberg framework to analyze business interactions between biofuels and food crop producers⁴. Yue and You propose a mixed integer nonlinear programming (MINLP) Stackelberg framework for the

optimal design of multi-echelon supply chain systems⁵. Zamarripa *et al* solve cooperative and competitive supply chain planning problems using a framework in which Nash equilibria are identified through the assembly of a payoff matrix^{7,8,15}. Game theory has seen use in engineering design problems as well: Bard, Plummer, and Sourie use a Stackelberg game framework to determine optimal biofuel tax credit policies⁹. Pierru uses Aumann-Shapley cost sharing to allocate refinery carbon dioxide emissions over finished products¹⁰. Chew *et al* present a game theoretic model for water integration in industrial parks¹¹. Recent reviews of supply chain optimization literature by Papageorgiou¹² and Sahebi, Nickel, and Ashayeri¹³ identify few papers taking advantage of game theoretic principles, and of the research identified in those reviews and here, none make use of dynamic game theoretic models. The model used by Tominac and Mahalec¹, for example, is a multiperiod planning problem formulated under a static Cournot game objective, precluding the ability of competitors to react to temporally changing market conditions.

Dynamic game theoretic problems are problems of optimal control, and have seen extensive research and use in the area of process control¹⁴. Due to the breadth of the field of dynamic (or differential) game theory, we restrict ourselves to a review of theory regarding Cournot oligopoly models. Dynamic Cournot oligopoly models owe their present day form to developments made by Simaan and Takayama, whose pricing model is based on an adjustment function relating price at some instant to all historical prices¹⁵. Fershtman and Kamien presented a variation on this model in which the rate of price adjustment is allowed to take an arbitrary value; their model is referred to as the Cournot oligopoly with sticky prices¹⁶. In their subsequent paper, equilibrium price trajectories are analyzed on finite time horizons, and are shown to exhibit asymptotic steady state (or turnpike) properties in which the equilibrium trajectory

approaches the infinite horizon steady state price trajectory from the outset of the time horizon, but deviates as the game terminates¹⁷. Fershtman and Kamien examine primarily closed loop equilibrium trajectories representative of subgame perfect Nash equilibria; Cellini and Lambertini examine open loop equilibrium steady state properties¹⁸, and steady state properties of both open and closed loop strategies in oligopoly games with product differentiation¹⁹. Wiszniewska-Matyszkiew, Bodnar, and Mirota extend the analysis of the dynamic Cournot oligopoly to off-steady states²⁰. These dynamic oligopoly models assume that competitors' individual actions result in significant impact on the system. Wiszniewska-Matyszkiew presents dynamic game models, in both discrete²¹ and continuous²² time domains, involving competitor continua of sizes large enough to render an individual's influence negligible. Such frameworks are important in the analysis of renewable resource usage. Zazo *et al* analyze a number of constrained dynamic game models as potential games²³.

Taking advantage of the dynamic Cournot oligopoly structure, we propose a dynamic closed-loop game theoretic process plant production planning framework based on a discrete time potential game formulation. To the best of our knowledge, such a framework has not been proposed within the body of literature of engineering supply chain planning. The closed loop dynamic game formulation allows competitors to respond to each other temporally, reacting to competing players' decisions at prior time points¹⁶. This dynamic framework synergizes with multiperiod strategic and tactical production planning models; planning decisions at each period are also interpreted as competitive best responses to decisions from prior periods. Competitor decision variables in this framework are infinitely valued, assuring from a theoretical standpoint that at least one Nash equilibrium strategy exists^{24,25,26}. A Nash equilibrium trajectory to a dynamic potential game is defined as the maximum of the dynamic potential function; thus the

proposed framework guarantees an equilibrium solution exists and is obtainable using numerical optimization²⁷. The properties of this dynamic potential game framework are illustrated using a set of competing petrochemical refiners with a focus on the interpretation of the competitor elimination scenario. The novel elements presented in this work are:

- A dynamic potential game theoretic framework for production planning problems yielding Nash equilibrium trajectories;
- The derivation of the dynamic potential function corresponding to the modified Cournot oligopoly model of Tominac and Mahalec¹ and the demonstration of the validity of this potential function;
- A case study in which a refiner threatened with closure predicts competitor hostility throughout phases of vulnerable growth and expansion in order to determine whether the threat of closure is legitimate and if so, whether closure can be prevented through upgrading of its facilities;
- If facility upgrading results in this framework, the time available to complete the upgrades – beyond which closure occurs – is also determined.

Background

Dynamic Nash equilibrium

The dynamic Nash equilibrium is defined in the same way as the static Nash equilibrium: for a set of players R playing a competitive game G over N discrete periods of time and having available strategies $\xi_{nr} \in \Xi_{nr}$ (where ξ and Ξ are used to denote unique strategies and strategy sets, respectively) with objective functions $J_r(\xi_{rn}, \xi_{-rn})$ of both each player's own and opposing players' strategies (denoted $-r$), then the set of Nash equilibrium strategy trajectories (denoted

as player-indexed vectors $\bar{\xi}_r = [\xi_{r1}, \dots, \xi_{rN}]$ to the game, $G\{\bar{\xi}_1^*, \dots, \bar{\xi}_R^*\}$ have the property in Eq. (1)¹⁷.

$$J_r(\bar{\xi}_r, \bar{\xi}_{-r}^*) \leq J_r(\bar{\xi}_r^*, \bar{\xi}_{-r}^*) \quad \forall r \in R, \bar{\xi}_r \in \bar{E}_r$$

(1)

There exist several important distinctions between dynamic and static Nash equilibrium, and we restrict ourselves to a discussion of these as they relate to the Cournot oligopoly with sticky prices. The dynamic closed-loop Nash equilibrium trajectory steady state value in games of infinite duration is not equivalent to the corresponding static game Nash equilibrium value¹⁶. Closed-loop dynamic games of finite duration exhibit a turnpike property prior to termination, but will exhibit an intermediate steady state value at some point following the beginning of a sufficiently long time horizon and before its end; this intermediate steady state is not equivalent to the corresponding static game Nash equilibrium¹⁷. As the number of firms increases to infinity, then the steady state values of these games approach the static Nash equilibrium values, representing a dynamic perfectly competitive environment¹⁸. Games of finite duration terminate at the static game Nash equilibrium; players become perfectly competitive when there is no future time in which to compete^{18,28}. While the equilibrium steady state value is unique, the strategy trajectories which yield this value are generally not unique¹⁷.

Problem statement

We examine a dynamic version of the game theoretic refinery production planning problem presented by Tominac and Mahalec¹, in which multiple refineries are considered in a framework in which each refinery is a private firm seeking to maximize its profits. Refiners are rational in the game theoretic sense and are competitive. All refiners are capable of producing

identical products and have access to the same set of crude feedstocks, but refineries vary in size and capacity. Geographic regions of sale are identified as markets in which each product has a nominal demand level and a price determined by a modified linear Cournot model¹. Markets containing refineries are identified as domestic markets and receive all products from those refiners, who are in turn obligated to supply domestic markets with product levels within some predefined socially acceptable window, and thus also a predefined price window. The potential for domestic market infeasibility is avoided by allowing domestic market refiners to import finished product at a fixed cost from other refiner-owned assets. Markets lacking any local refiner presence are defined as global markets, and are connected to domestic markets by pipelines. All refiners in connected domestic markets may export to global markets without any restrictions on total market supply, or thus price. In all cases, markets are assumed to clear at the Cournot price corresponding to total market supply in a given period of time. The dynamic Cournot oligopoly pricing model is such that price in a given period of time is a function of the initial product price and all prior supply as well as the product supplied within the current period. Under these conditions refiners are faced with the following problem:

- To determine the product amounts which must be provided and the delivery time of those products at the domestic market level in order to satisfy supply obligations together with other competing domestic market refiners, including whether and when any product imports are required (a strategic decision).
- To determine the timing and amounts of each product for export to available global markets in which a refiner is competing with both local domestic refiners and refiners in other domestic markets (a strategic decision).

- To determine the required amounts of available crude stocks and purchase timing, and the blend plan to be used to produce products in the means yielding the greatest profit (production planning decisions).

We focus our attention on the scenario in which a high-cost refiner may be eliminated from a domestic market and examine this problem in the dynamic game theoretic production planning framework. This scenario is based on the notion that market prices are higher while the high-cost refiner is active, allowing all refiners to benefit. The remaining low-cost refiners may unanimously elect to shut down the high-cost refiner, at which point they gain the high-cost refiner's market share. The problem is interpreted as a trade-off between higher prices and a larger market from the low-cost refiners' perspective; Tominac and Mahalec¹ discuss the ramifications of this interpretation of the problem in depth, which is based on observations of refining assets and pipeline networks in western Canada²⁹.

We consider instead the high-cost refiner's perspective on the problem, which is inherently one of survival. In a dynamic game theoretic framework, the high-cost refiner may be shut down in any period. From the perspective of the low-cost refiners, this will occur when and if it maximizes their own profits to do so. The high-cost refiner does not want to close, but is limited to making production decisions as if it were not facing this problem; i.e., production levels for the high-cost refiner are determined from the equivalent dynamic potential game with no elimination option. We consider the transition of a high-cost refiner to a low-cost refiner through an implicit upgrade argument: as the high-cost refiner implements upgrades, the price increases associated with its presence diminish in scale. If the high-cost refiner can implement upgrades which reduce its associated price increases to zero, then it is considered to have become competitive with the low-cost refiners (it has in effect become a low-cost refiner) and cannot be

eliminated by them any longer. If the high-cost refiner is unable to complete upgrades in this manner, then it is shut down. The assumptions made regarding the high-cost refiner's problem are as follows:

- A high-cost refiner may become a low-cost refiner through upgrading.
- High-cost refiners are unable to import product from other assets; at some time after transitioning to low-cost refiner status and not threatened with elimination, a high-cost refiner will establish an import source.
- Low-cost refiners cannot be eliminated from markets.
- The elimination decision is made unanimously by the set of low-cost refiners; this is in part to avoid gaming effects on that decision³⁰.
- A high-cost refiner is vulnerable until it becomes a low-cost refiner; i.e., it is vulnerable until its upgrades are complete.
- The upgrade process may be examined purely through the high-cost refiner's changes to its required price increase.
- Multiple upgrade stages may be examined by sequential solution of the dynamic model using changing values of the price increase and setting the initial price to the current period price in which an upgrade is completed.
- If the high-cost refiner is not eliminated within the finite horizon, it is never eliminated. The static game Nash equilibrium includes the high-cost refiner.

Under these assumptions, the high-cost refiner's problem is to determine if an upgrade plan is available which results in its continued participation in the market as either a high-cost or low-cost refiner. This is a strategic decision on the high-cost refiner's part, and is a function of the competing low-cost refiners' profits, not its own. We examine a scenario with three refiners,

two of which are low-cost and one high-cost, all operating out of a single domestic market which has export access to one global market. All refiners in this scenario produce two products: regular gasoline and type 2 consumer diesel. This scenario is laid out in Figure 1.

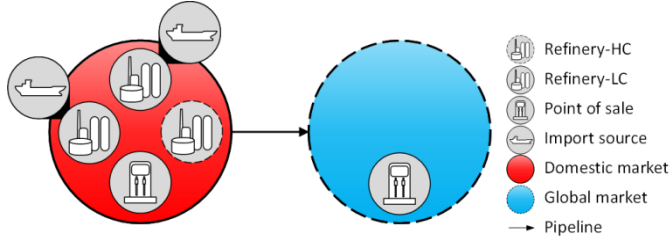


Figure 1. Scenario layout

Models and Formulation

Deriving a dynamic potential function

The basis of the discrete dynamic Cournot oligopoly model is that the price of a homogenous product in a period of time is equal to a weighted combination of its price in the previous period of time and the corresponding static Cournot oligopoly price in the current period. Price is represented as π_{npw} , static Cournot price is $\tilde{\pi}_{npw}$, and the weighting factor is a scalar s with value between zero and one inclusive as in Eq. (2).

$$\pi_{npw} = (1 - s)\tilde{\pi}_{npw} + s\pi_{(n-1)pw}$$

(2)

The static price term represents a linear Cournot pricing model. We use the demand-based model of Tominac and Mahalec¹ in Eq. (3), where A_{pw} and B_{pw} represent the initial price of a product p in market w and the price corresponding to a supply level of D_{pw} , respectively, and where q_{nrpw} is the amount of product p supplied by refiner r to market w in period n .

$$\tilde{\pi}_{npw} = A_{pw} + B_{pw} - \frac{A_{pw}}{D_{pw}} \sum_r q_{nrpw}$$

(3)

Price π_{npw} is shown in its general recursive expression in Eq. (4), and with $\tilde{\pi}_{npw}$ expanded in Eq. (5). The dependence of price on the initial price π_{0pw} is apparent in the recursive expression; it is assumed to be $\pi_{0pw} = A_{pw} + B_{pw}$.

$$\pi_{npw} = \sum_{n'=1}^n [s^{n-n'}(1-s)\tilde{\pi}_{n'pw}] + s^n \pi_{0pw}$$

(4)

$$\pi_{npw} = \sum_{n'=1}^n \left[s^{n-n'}(1-s) \left(A_{pw} + B_{pw} - \frac{A_{pw}}{D_{pw}} \sum_r q_{n'rpw} \right) \right] + s^n \pi_{0pw}$$

(5)

We assume a general refiner cost function of the form $C_{nr}(q_{nrpw})$ such that refiner profit J_r may be defined as in Eq. (6), and expressed as in Eq. (7). The parameter e^{-in} is a discounting factor decreasing the weight of future profits using the parameter i to determine the amount.

$$J_r = \sum_n e^{-in} \left(\sum_p \sum_w [\pi_{npw} q_{nrpw}] - C_{nr}(q_{nrpw}) \right)$$

(6)

$$J_r = \sum_n e^{-in} \left(\sum_p \sum_w \left[\left(\sum_{n'=1}^n [s^{n-n'}(1-s)\tilde{\pi}_{n'pw}] + s^n \pi_{0pw} \right) q_{nrpw} \right] - C_{nr}(q_{nrpw}) \right)$$

(7)

The potential function is defined according to the methods defined by Monderer and Shapley²⁷ and Slade^{31,32} and is defined by Eq. (8). We separate the usual Ω into $\Omega_r^S + \Omega_r^C$ to

isolate costs from the main part of the term. The definitions of Ψ , Ω_r^S , and Ω_r^C are provided in Eqs. (9), (10), and (11).

$$Z = \Psi + \sum_r (\Omega_r^S + \Omega_r^C) \quad (8)$$

$$\Psi = \sum_p \sum_w \sum_n \sum_{n'=1}^n \left[e^{-in} s^{n-n'} (1-s) \left(-\frac{A_{pw}}{D_{pw}} \right) \left(\sum_r \sum_{\substack{r' \\ r' \neq r}} q_{nrpw} q_{n'r'pw} \right) \right]$$

$$\Omega_r^S = \sum_p \sum_w \sum_n \left(\sum_{n'=1}^n \left[e^{-in} s^{n-n'} (1-s) \left((A_{pw} + B_{pw}) q_{nrpw} - \frac{A_{pw}}{D_{pw}} q_{nrpw} q_{n'rpw} \right) \right] + s^n \pi_{0pw} q_{nrpw} \right) \quad (9)$$

$$\Omega_r^C = - \sum_n e^{-in} C_{nr} (q_{nrpw}) \quad (10)$$

(11)

Verification of the dynamic potential function

The dynamic potential function defined by Eqs. (8), (9), (10), and (11) is verified as a potential function for the dynamic game by demonstration that it simultaneously maximizes the profits of all competing refiners $r \in R$ ^{31,32}. The condition that must be satisfied by the potential function is defined in Eq. (12). The derivative of profit J_r is defined in Eq. (13).

$$\frac{\partial Z}{\partial q_{nrpw}} = \frac{\partial J_r}{\partial q_{nrpw}} \quad \forall n \in N, r \in R, p \in P, w \in W \quad (12)$$

$$\begin{aligned} \frac{\partial J_r}{\partial q_{nrpw}} = e^{-in} & \left\{ \sum_{n'=1}^n \left[s^{n-n'} (1-s) \left(A_{pw} + B_{pw} - \frac{A_{pw}}{D_{pw}} \sum_{r'} q_{n'r'pw} \right) \right] \right. \\ & \left. + (1-s) \left(-\frac{A_{pw}}{D_{pw}} q_{nrpw} \right) + s^n \pi_{0pw} - \frac{\partial C_{nr}(q_{nrpw})}{\partial q_{nrpw}} \right\} \end{aligned}$$

(13)

For the dynamic potential function derivatives for each component are determined to be as in Eqs. (14), (15), and (16).

$$\frac{\partial \Psi}{\partial q_{nrpw}} = e^{-in} \sum_{n'=1}^n \left[s^{n-n'} (1-s) \left(-\frac{A_{pw}}{D_{pw}} \sum_{\substack{r' \\ r' \neq r}} q_{n'r'pw} \right) \right]$$

(14)

$$\begin{aligned} \frac{\partial \Omega_r^S}{\partial q_{nrpw}} = e^{-in} & \left(\sum_{n'=1}^n \left[s^{n-n'} (1-s) \left(A_{pw} + B_{pw} - \frac{A_{pw}}{D_{pw}} q_{n'r'pw} \right) \right] + (1-s) \left(-\frac{A_{pw}}{D_{pw}} q_{nrpw} \right) \right. \\ & \left. + s^n \pi_{0pw} \right) \end{aligned}$$

(15)

$$\frac{\partial \Omega_r^S}{\partial q_{nrpw}} = -\frac{\partial C_{nr}(q_{nrpw})}{\partial q_{nrpw}}$$

(16)

Addition of the dynamic potential function derivatives yields the profit function derivative in Eq. (13), and satisfies the condition in Eq. (12); therefore the discretized dynamic potential function is correct.

A potential function for the elimination game

The competitor elimination scenario uses the definition of $\tilde{\pi}_{npw}$ in Eq. (17) which incorporates the parameter A_{pw}^{HC} and binary variable y_{nw} in order to increase market prices when the high-cost refiner is active. The high cost refiner is defined to be active when the value of y_{nw} is one, and inactive when it is zero. The variable y_{nw} is constrained such that it is fixed to zero following the first period in which it becomes zero; thus the high-cost refiner is never allowed back into operation once it has been eliminated. Since the high-cost refiner is assumed to active at the zeroth period, initial price is taken as $\pi_{0pw} = A_{pw} + B_{pw} + A_{pw}^{HC}$.

$$\tilde{\pi}_{npw} = A_{pw} + B_{pw} + A_{pw}^{HC} \sum_{w' \in RW_D} (y_{nw'}) - \frac{A_{pw}}{D_{pw}} \sum_r q_{nrpw}$$

(17)

The potential function for the competitor elimination game taking this change into account is defined by Eqs. (18), (19), (20), and (21). The demonstration that this potential function is also correct follows the same logic as the general case.

$$Z = \Psi + \sum_r (\Omega_r^S + \Omega_r^C)$$

(18)

$$\Psi = \sum_p \sum_w \sum_n \sum_{n'=1}^n \left[e^{-in} s^{n-n'} (1-s) \left(-\frac{A_{pw}}{D_{pw}} \right) \left(\sum_r \sum_{\substack{r' \\ r' \neq r}} q_{nrpw} q_{n'r'pw} \right) \right]$$

(19)

$$\Omega_r^S = \sum_p \sum_w \sum_n \left(\sum_{n'=1}^n \left[e^{-in} s^{n-n'} (1-s) \left(\left(A_{pw} + B_{pw} + A_{pw}^{HC} \sum_{w' \in RW_D} y_{nw'} \right) q_{nrpw} - \frac{A_{pw}}{D_{pw}} q_{nrpw} q_{n'rpw} \right) + s^n \pi_{0pw} q_{nrpw} \right] \right)$$

(20)

$$\Omega_r^C = - \sum_n e^{-in} C_{nr} (q_{nrpw})$$

(21)

Introduction of the binary variable y_{nw} yields a bilinear term of the form $y_{nw} q_{nrpw}$ for which there exists an exact linearization³³. We replace the binary term identified in Eq. (22) with the equivalent linearization, and add the constraints in Eqs. (23), (24), and (25) to the model to preserve the original functionality. The parameter \bar{q}_{nrpw} is used to represent the maximum value of q_{nrpw} .

$$\sum_{w' \in RW_D} y_{nw'} q_{nrpw} \equiv \sum_{w' \in RW_D} \gamma_{nrpww'}$$

(22)

$$\gamma_{nrpww'} + \gamma_{nrpww'}^* = q_{nrpw} \quad \forall n \in N, r \in R, p \in P, (w, w') \in W$$

(23)

$$\gamma_{nrpww'} \leq y_{nw'} \bar{q}_{nrpw} \quad \forall n \in N, r \in R, p \in P, (w, w') \in W$$

(24)

$$\gamma_{nrpww'}^* \leq (1 - y_{nw'}) \bar{q}_{nrpw} \quad \forall n \in N, r \in R, p \in P, (w, w') \in W$$

(25)

The potential function in the elimination scenario takes into account the high-cost refiner only in the Ψ term; it is ignored in the addition of both Ω_r terms. This formulation presents an interesting interpretation of the scenario: the high-cost refiner examines when the low-cost refiners prefer to have it active in the petroleum market and when or if they would prefer to have it inactive. Its own profits do not factor into the decision, which might be anticipated given the assumed game theoretic definitions of rationality.

Solution Procedure

Since the high-cost refiner's profits are not factored into the elimination scenario potential function defined by Eqs. (18) to (21) this potential function cannot be used to determine the high-cost refiner's production plan. We thus use a two stage solution process with the rationale that the high-cost refiner selects as its production plan the game theoretically optimal scheme as though it is not facing elimination (i.e., its competitive production scheme) and then holds to that production scheme in the elimination scenario. Mathematically, this corresponds to first solving the potential function corresponding to Eqs. (8) to (11) to calculate the high-cost refiner's Nash equilibrium profile, and then holding these values constant for only the high-cost refiner in the solution of the elimination scenario potential function defined by Eqs. (18) to (21). Thus the low-cost refiners may alter their own production schemes in response to the high cost refiner's fixed choice, but *ceteris paribus* it is anticipated that any changes in behaviour from the first stage to the second are attributable only to whether and when the high-cost refiner is eliminated.

Refinery Models

The refinery production planning model implemented in this work is that of Tominac and Mahalec¹ defined in the supplementary material. We make one change to the model in that we do not limit the high-cost refiner's production throughput to a fixed value in the elimination scenario. We use only the regular gasoline and type 2 diesel products in this model for two reasons: Tominac and Mahalec¹ observed mainly small values of the other four products whose production they modelled; and to reduce the overall model size and complexity in the dynamic framework.

Results and Discussion

On avoiding elimination

Results are interpreted in the context of the high-cost refiner and its objective of remaining an active market participant. To reiterate, we assume that the high-cost refiner is not willing to modify its current market share in order to disincentivize its own elimination. Instead, the high-cost refiner's survival is predicated on the amount by which it maintains higher prices than would exist following its elimination, versus the excess profit that the low-cost refiners would accrue through the acquisition of its market share. As the high-cost refiner attempts to upgrade its processes and reduce its costs, the amount by which it requires high prices is reduced, and the market prices fall to match. The interaction between prices and competitors is complex in real cases; we attempt to isolate price changes in the facet of upgrades through the notion of the high-cost refiner's higher price requirement.

Using the standard data set to construct a dynamic game theoretic refinery production planning model over an eight month time horizon discretized into ten periods, the Nash

equilibrium product price trajectories in Figure 2 are obtained. The standard data set of Tominac and Mahalec¹ assumes a value of A_{pw}^{HC} equal to 5% of A_{pw} in domestic markets, and zero elsewhere. In this example the high-cost refiner is eliminated in the fourth period; the price trajectories corresponding to the case in which no elimination occurs are included in the figure for comparison. Domestic market product prices actually rise following elimination due to an overall decrease in market supply. Figure 3 presents refiner and market supply volume trajectories for domestic regular gasoline sales. Market supply opens at its upper limit and remains there for three periods; following elimination of the high-cost refiner market supply drops below its upper limit, causing prices to rise in subsequent periods, though both low-cost refiners are able to increase their production rates. Since the high-cost refiner is able to export, this change influences global market prices as well. Thus while prices are high low-cost refiners prefer the additional marginal profit obtained by allowing the high-cost refiner to persist. As prices drop, they prefer the market share. In this example, the high-cost refiner thus has three periods in which to implement upgrades that would enable it to compete with the low-cost refiners and prevent its elimination in period four. As this corresponds to less than three months of time, it is unlikely the high-cost refiner would be able to prevent this outcome.

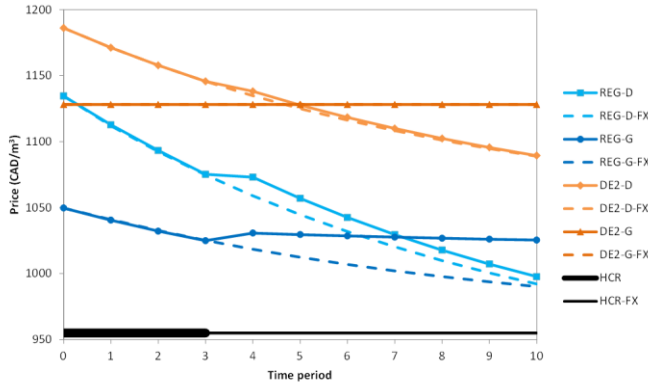


Figure 2. Nash equilibrium price trajectories with normal high-cost refiner A_{pw}^{HC} value. REG and DE2 are used to denote regular gasoline and diesel prices; D and G are used to denote domestic and global markets. The line labelled HCR indicates whether and when the high-cost refiner has been eliminated. The dashed FX lines indicate price trajectories in the case no elimination occurs.

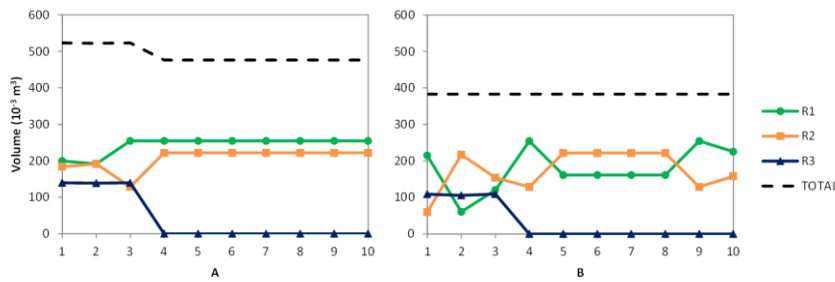


Figure 3. Domestic market supply of regular gasoline including (A) and excluding (B) imports of finished product.

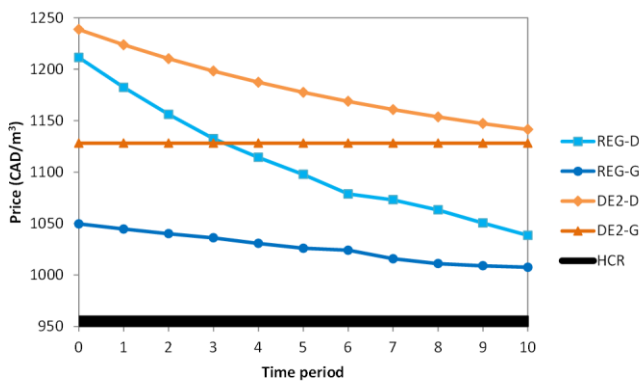


Figure 4. Nash equilibrium price trajectories with large high-cost refiner A_{pw}^{HC} value. REG and DE2 are used to denote regular gasoline and diesel prices; D and G are used to denote

domestic and global markets. The line labelled HCR indicates whether and when the high-cost refiner has been eliminated.

A second scenario is examined in which the high-cost refiner is extremely inefficient and requires market prices to be in significant excess of their competitive values. This scenario is represented using A_{pw}^{HC} equal to A_{pw} in domestic markets. The Nash equilibrium price trajectories are presented in Figure 4. In this scenario the high-cost refiner is never eliminated; even as prices decrease from their initial values. Since the high-cost refiner is active in the terminal period, we interpret this scenario to represent a market in which the high-cost refiner is never eliminated; the low-cost refiners desire it to remain active indefinitely. Despite its inefficiencies there is thus no external incentive to the high-cost refiner to upgrade its facilities. Optimization of its own internal costs independent of the greater market context in which it operates may suggest capacity expansion as a viable and profitable endeavor for the operators of this refinery; nevertheless they run the risk of becoming too competitive. If the high-cost refiner in this scenario upgrades its facilities (following which we assume that market prices decrease via A_{pw}^{HC} adapt to the new market context) it may find itself in the initial scenario presented wherein its benefit to its competitors has decreased below the value of its market share in a market that no longer includes itself, and is subsequently eliminated by its competitors. The high-cost refiner thus must be able to upgrade its facilities to the extent that it becomes a competitive low-cost refiner, or upgrade only so far as to improve its operating costs without risking elimination. Either of these options entails careful evaluation of the greater market context during and following its upgrade procedures. Of course, in this case the high-cost refiner may elect to do nothing at all and anticipate its continued market participation; any risk of elimination emerges only when and if it takes action to improve its efficiency.

Influence of time discretization

Dynamic games are differential optimal control problems; discretization of the differential form of the dynamic game facilitates the use of NLP and MINLP solvers in the solution of games of finite length. The formulation of the dynamic Cournot game is such that the terminal period of the dynamic game is interpreted as an equivalent static Cournot game; i.e., the terminal point of the finite dynamic Cournot game Nash equilibrium trajectory is the Nash equilibrium of a static Cournot game. A fixed length time horizon of eight months is considered in this work: thus changing the number of discretization points (or time periods) in the dynamic model changes the duration of each period, and by the interpretation of discrete dynamic games, the static game solution corresponding to the terminal period. The terminal point of a Nash equilibrium trajectory is not expected to remain constant in alternative time discretizations; it is of interest whether the quantitative pattern observed over the horizon is altered by the selection of a different time discretization as well.

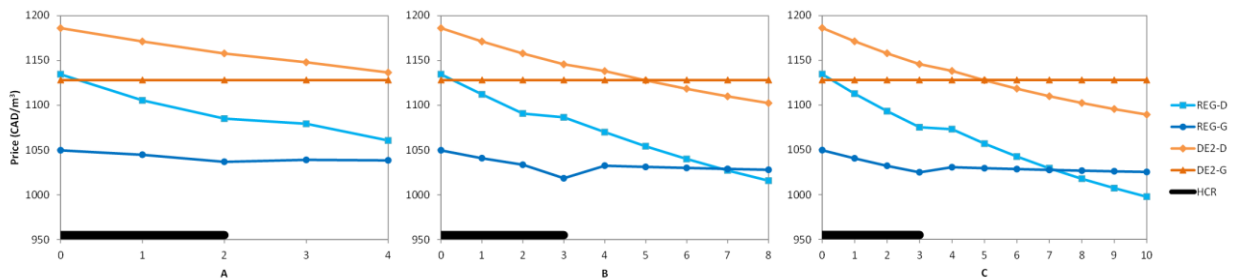


Figure 5. Nash equilibrium price trajectories obtained for discretizations of four (A) eight (B) and ten (C) periods of an eight month planning horizon.

Maintaining the horizon length of eight months, discretizations of four, eight, and ten time periods are applied to the dynamic production planning model and the Nash equilibrium price trajectories obtained are presented in Figure 5. It is observed that the terminal points of the trajectories vary with time discretization as anticipated. The qualitative paths of the trajectories

are similar in each case, though in the case of the four point discretization we observe that resolution is lost. Importantly, the elimination trend does not seem to be adversely impacted by the selection of horizon discretization; it is interpreted from this that the elimination decision is a function of profit and the total duration of the time horizon, but not discretization.

Existence of multiple equilibria

While price trajectories represent (generally) unique dynamic Nash equilibrium solutions to the problems examined, the underlying refiner decision variables yielding these trajectories are not unique. Refiner in-house production levels (i.e., excluding imports) of domestic market regular gasoline are presented in Figure 3B. The production trajectories corresponding to the refiners are linked: even excluding the import volumes, the collective regular gasoline total is the same at every point on the horizon.

The game constraints limit the refiners' strategy space and force them to operate in concert with one another. The resulting solution is a generalized Nash equilibrium^{34,35,36}. The existence of non-unique solutions is addressed by Rosen through the process of normalization³⁷. We do not attempt to apply normalization procedures to our solutions; we are interested in the macroscopic behaviour of the refiners: knowing that the underlying planning model is feasible (and that multiple solutions exist) demonstrates the connections between the physical capabilities of the plants and organization-level decision making. Since the optimum is identical in these equivalent cases, the qualitative and quantitative interpretations of the results with regard to strategy are not altered. Proper application of normalization would presumably yield smooth in-house as well as import trajectories for individual refiners, which would be desirable from an implementation perspective.

Model solution statistics

Model size is a particular challenge in the solution of discretized dynamic game theoretic problems; the growth of bilinear terms in the presented formulation increases rapidly with each additional time point added to the horizon. Price at any point in time is a function of all preceding prices and the original price at time zero; the original price is asymptotically rejected in successive time points, but nevertheless complicates the model equations. The solution procedure defined for this framework applies CONOPT 3.17A³⁸, IPOPT 3.12³⁹, and ANTIGONE 1.1⁴⁰ in the first stage NLP to determine the high-cost refiner's production trajectory, and then DICOPT⁴¹ and ANTIGONE 1.1 to solve the MINLP which determines whether and when the high-cost refiner is eliminated. Solution statistics for the models presented in this work are collected in Table 1, and were generated on a Dell Optiplex 9010 computer with Intel Core-i7-3770 CPU and a 3.40 GHz processor operating on Windows 10 64-bit system.

Table 1. Solution statistics

	Standard 4 period	Standard 8 period	Standard 10 period	Inefficient 10 period
Model statistics (GAMS)				
Single equations	2732	5802	7487	7487
Single variables	2042	2534	5972	5972
Nonlinear entities	304	1120	1720	1720
Second Stage ANTIGONE Output				
Solve summary				
Objective value	764.7155	612.1062	549.6476	772.8618
Model status	1	1	1	1
After pre-processing				
Variables	579	1263	1653	1653
Continuous	575	1255	1643	1643
Binary	4	8	10	10
Equations	1122	2484	3267	3267
Linear	969	1931	2418	2418
Nonconvex nonlinear	153	553	849	849

Nonlinear terms	1264	5216	8200	8200
Bilinear/quadratic	1232	5152	8120	8120
Sigmoidal	32	64	80	80
Solve statistics				
Relative gap	1.00E-09	1.00E-09	1.00E-09	1.00E-09
Total time (CPU s)	4.25	23.46	57.95	61.69
Total combined first and second stage solve time (CPU s)				
	1212.10	184.27	1021.48	555.16

Data are presented for the second stage ANTIGONE solve. The numbers of bilinear terms increase quickly with the number of time periods, and results in large nonconvex nonlinear problems overall. In all four cases, global optimality is obtained, indicated by the model status of 1, with a relative optimality gap of 1×10^{-9} . The solution times are on the scale of seconds to minutes for the final second stage solve; more time is spent obtaining feasible solutions in the first stage problem and in the DICOPT phase of the second stage. The multistage solution approach is such that the solution at which ANTIGONE starts its search possesses an optimality gap within stopping tolerance. Once ANTIGONE initializes the solution and completes its pre-solve operations, only a single iteration is required before the algorithm concludes optimality has been obtained. Without the two stage approach, a global solver has yet to be found which closes the gap in reasonable time.

Conclusions

A framework has been presented for dynamic game theoretic refinery production planning problems in which refiners operate in multiple markets with pricing functions based on a modified dynamic Cournot oligopoly model with sticky prices. This framework is used to examine competitor elimination scenarios from the perspective of a refiner facing potential elimination by its competitors. Our results indicate that there exist scenarios in which a refiner's

best strategy is to upgrade its facilities in order to avoid elimination, while in other cases the same refiner would be better off taking no action at all. Where upgrades are required in order to avoid closure, the available time before closure is obtained and presents a deadline on upgrade procedures. These decisions are the result of a complex trade-off between a refiner's own cost structures and its interactions with its competitors. Planning scenarios accounting for these types of interactions have not been examined in prior production planning literature, and can identify market forces whose impacts are otherwise not possible to quantify in an optimal planning framework. Process plants in large markets may face these complex types of decisions wherein internal costs and planning have market-wide impacts and consequences; the presented game theoretic planning framework aids in exploring these problems.

Notation

Sets

N	(n) time periods
R	(r) refiners
R_L	(r) low-cost refiners
R_H	(r) high-cost refiners
P	(p) products
W	(w) markets
W_D	(w) domestic markets
W_G	(w) global markets
RW_D	(r, w) refiner r is located in domestic market w
Ξ	(ξ) strategy set

Parameters

s	Price stickiness factor
A_{pw}	Price decline rate
A_{pw}^{HC}	Price increase due to high-cost refiner activity
B_{pw}	Price of product p in market w corresponding to supply of exactly D_{pw}
D_{pw}	Nominal market supply of product p in market w
π_{0pw}	Initial price of product p in market w corresponding to the zeroth time period
i	Profit discounting factor
\bar{q}_{nrpw}	Maximum production of product p in period n to market w by refiner r including both in-house production and imports

Continuous Variables

J_r	Profit for refiner r
π_{npw}	Price of product p in market w in period n
$\tilde{\pi}_{npw}$	Static Cournot price of product p in market w in period n
q_{nrpw}	Total volume of product p delivered to market w in period n by refiner r accounting for in-house production and imports
C_{nr}	General cost function of refiner r in period n
Z	Potential function value
Ω_r	Potential function individual term
Ω_r^S	Profit component of Ω_r
Ω_r^C	Cost component of Ω_r

Ψ Potential function collective term
 $\gamma_{nrpww'}$ Exact bilinear reformulation variable

Binary Variables

y_{nw} Determines at which period n high-cost refiners are eliminated from market w

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